Exercises

Spontaneous Symmetry Breaking and Field Theory 1

Sheet 4

4/1 Ornstein-Zernicke correlation function

Calculate for a structurally homogeneous system the correlation function $G(\mathbf{r},\mathbf{r}') = G(\mathbf{r}-\mathbf{r}')$ by means of the fluctuation-dissipation theorem for systems in an inhomogeneous magnetic field

$$\mathbf{H}(\mathbf{r}) = \begin{pmatrix} 0 \\ 0 \\ H + \delta H(\mathbf{r}) \end{pmatrix}, \tag{1}$$

with zero volume average of $\delta H(\mathbf{r}) = \int d^3r'\ \delta(\mathbf{r}-\mathbf{r}')\ \delta H(\mathbf{r}')$. The theorem reads

$$\delta\overline{\sigma(\mathbf{r})} = \overline{\sigma(\mathbf{r})} - \sigma = \frac{1}{kT}\int d^3r'\ G(\mathbf{r},\mathbf{r}')\ \delta H(\mathbf{r}'), \tag{2}$$

where σ is the volume average of $\overline{\sigma(\mathbf{r})}$.
<u>Hint:</u> Write down the Landau-Ginzburg equation for $\overline{\sigma(\mathbf{r})}$ by taking into account the gradient-term in the Ginzburg-Landau-Wilson functional. Subtract from this equation the volume-averaged equation to get an equation for $\delta\overline{\sigma(\mathbf{r})}$, in which you then insert eq. (2). Use Fourier-representations to solve the so derived equation for $G(\mathbf{r}-\mathbf{r}')$.

4/2 Homogeneous functions

A homogeneous function of one variable λ is defined by the relation $F(\lambda X) = g(\lambda)F(X)$ for arbitrary real λ. Show that $g(\lambda) = \lambda^p$ with a real number p. The generalization to two variables is $F(\lambda X, \lambda Y) = \lambda^p F(X,Y)$.
<u>Hint:</u> Consider $F((\mu\lambda)X) = g(\mu\lambda)F(X)$, and derive a differential equation for $g(\lambda)$ by arguments similar to those used in thermodynamics to derive the Gibbs-Duhem relation.

The homogeneity postulate for the singular part g_{sing} of the Gibbs free energy may be written in the general form

$$g^{(1)}_{\text{sing}}(L^y t, L^x H) = L^d g^{(1)}_{\text{sing}}(t,H) \tag{3}$$

with real values of y and x and with the spatial dimension d for arbitrary values of L. Show the equivalence of (3) with the following forms:

1.

$$g^{(2)}_{\text{sing}}(\lambda^a t, \lambda^b H) = \lambda g^{(2)}_{\text{sing}}(t,H), \tag{4}$$

2.

$$g^{(3)}_{\text{sing}}(t,H) = t^p g^{(3)}_{\text{sing}}\left(\frac{H}{t^q}\right). \tag{5}$$

Thereby calculate the relations between (x,y), (a,b) and (p,q). Express the critical exponents α, β, γ and δ by x and y.